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ON QUASI-CHARACTERISTIC EQUATIONS AND SYSTEMS FOR REAL DIFFERENTIAL EQUATION

Jelena Vujaković*, **Miloje Rajović** **

Abstract. In linear homogenous differential equation of second order with constant coefficients, implementation of substitution $y=\exp(rx)$, where r is also constant, we obtain algebraic characteristic equation, which not only determines the type of solution-monotonous or oscillatory, but at the same time gives the solution itself. In this paper we shall prove that analogous principle applies even when coefficients in this equation are arbitrary continuously-differentiable functions. We will show that characteristic equation can also be implemented for non-constant coefficients and that the systems of those and quasi-characteristic equations are possible. This can be obtained by implementation of complex variable.

Keywords. Differential equation, a quasi-characteristic equation, a series-iteration method, Sturm's functions

Introduction and preliminaries

According to Liouville's principle it has been known that in the solutions of linear homogenous differential equation of second order the exponential functions $\exp(rx)$, $\exp(irx)$ are important. They can also be complicated and they give two general solution types: monotonous and oscillatory. Furthermore, in solutions are very frequent and product of type $y=uv$, where one multiplier is exponential function and other is some arbitrary function, but the point is that the both multiplies depend on constant coefficients.

For linear homogenous differential equations of second order

$$y''+a(x)y'+b(x)y=0 \tag{1.1}$$

with non-constant coefficients $a(x)$ and $b(x)$ we could find solution in complex form

$$y=f(x)\exp(ig(x)) \tag{1.2}$$

where $f(x)=f(a(x),b(x))$ and $g(x)=g(a(x),b(x))$. Substituting (1.2) and their derivates in equation (1.1) and then separating real and imaginary part we obtain system with respect to unknown $f(x)$ and $g(x)$, which we shall call system of quasi-characteristic equations for (1.1).

It is clear that the following lemma is valid